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PARAMETER ESTIMATION IN ACTIVE PLATE STRUCTURES USING GRADIENT OPTIMIZATION AND NEURAL NETWORKS

A. L. Araújo
ESTIG – Instituto Politécnico de Bragança
Campus de Sta. Apolónia, Apartado 134
5301-857 Bragança, Portugal
aaraujo@ipb.pt

C. M. Mota Soares
Instituto de Engenharia Mecânica – IST
Av. Rovisco Pais, 1049-001 Lisboa
cmmsoares@alfa.ist.utl.pt

J. Herskovits
COPPE – Universidade Federal do Rio de Janeiro, Caixa Postal 68503, 21945-970 Rio de Janeiro, Brazil
jose@optimize.ufrj.br

P. Pedersen
Department of Mechanical Engineering
Technical University of Denmark, Building 404
DK-2800, Kgs. Lyngby, Denmark
pauli@mek.dtu.dk

ABSTRACT

Two non-destructive methods for elastic and piezoelectric parameter estimation in active plate structures with surface bonded piezoelectric patches are presented.

The first of these solves the inverse problem through gradient based optimization techniques, minimizing the difference between experimental and numerical finite element eigenfrequencies for a test plate. Such minimization is conducted with FAIPA, a non linear interior point algorithm.

The second method relies on building a metamodel of the inverse problem, using artificial neural networks (ANN). The training data set is obtained through the same numerical model as in the first approach. The simulation of the network is then used with the experimental eigenfrequecy data set in order to produce an estimate for the material parameters.

Results from both approaches are compared and discussed through a simulated identification

INTRODUCTION

The need for accurate mechanical and piezoelectric parameters in modeling and analysis of active laminated plate type structures has become a major concern in active control applications. Traditional estimates based upon engineering tables provided by manufacturers are not always reliable for some of these applications, due to substantial variability among samples and dynamic ranges of interest and, more important, when such products are combined as components in an active composite material configuration, the effective values of such parameters are usually found to be quite different.

To address these issues we present two non destructive methods for parameter estimation in active plate structures with surface bonded piezoelectric patches and try to establish a comparison between the two approaches to the inverse problem: gradient based optimization techniques and metamodeling techniques, namely through the use of artificial neural networks.

In both techniques, the system response used was a set of undamped experimental free vibration natural frequencies. Thus, a numerical model capable of reproducing the response of the physical system is of paramount importance for both approaches. This numerical model is a finite element higher order laminated plate model that includes the piezoelectric effect [1].

This work is a generalization of previous works that used gradient optimization in single or multimaterial laminate configurations [2, 3, 4], and included identification of piezoelectric parameters [1], and it has the innovative aspect of applying ANN to estimate elastic and piezoelectric parameters in active laminates, using global response quantities such as natural frequencies of free vibration.

Elastic parameter estimation techniques have been proposed by several authors. An assessment of different approaches on eigenfrequency and optimization-based identification methods for estimation of mechanical properties on composite laminated plates is available in [5]. Other eigenfrequency-based methods for identification of elastic constants on laminated composite materials include methods based on response surfaces [6] and the use of model updating techniques [7]. Another class of inverse methods for estimation of elastic stiffness parameters in composite structures is based on ultrasonic and wave propagation measurements along with optimization techniques where, more recently, genetic algorithms have also been used [8, 9]. Artificial neural networks have also been used recently to inversely estimate elastic parameters of anisotropic laminated plates using surface displacement responses in a wave propagation simulation [10].

Regarding the estimation of both elastic and piezoelectric constants of surface bonded piezoelectric sensors and actuators in adaptive plate structures, gradient-based methods, among others, have been proposed [1, 11, 12].

NUMERICAL MODEL

The numerical model used is a higher order finite element laminated plate model with cubic expansion of the in-plane displacements in the thickness coordinate and constant transverse displacement through the thickness, as shown in Equation (1) for the laminated plate of Fig. 1.

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\theta_x(x,y,t) + \\ &\quad + z^2u_0^*(x,y,t) + z^3\theta_x^*(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\theta_y(x,y,t) + \\ &\quad + z^2v_0^*(x,y,t) + z^3\theta_y^*(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

In (1), u_0, v_0 and w_0 are the in-plane displacements in the *x*, *y*, and *z* directions, *t* is the time variable and θ_x and θ_y are the rotations of normals to the midplane about the *y* axis (anticlockwise) and *x* axis (clockwise), respectively. The functions u_0^*, v_0^*, θ_x^* and θ_y^* are higher order terms in the Taylor series expansion, defined also in the midplane of the plate.

Full details regarding the model development and implementation for dynamics can be found in [13], while its extension to account for the piezoelectric effect is described in detail in [1].

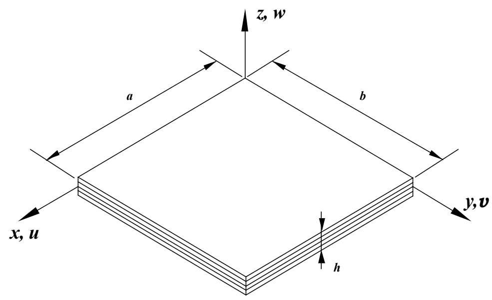

Figure 1. Laminated plate in global reference system

For each lamina (Fig. 2) in the laminate, the constitutive equations are represented in Equation (2), in the local principal material axes [14, 15].

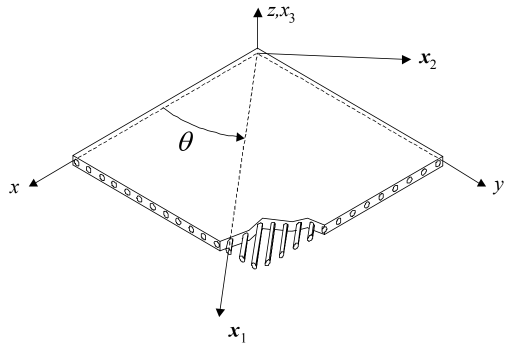

Figure 2. Orthotropic material lamina

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{Q}\boldsymbol{\varepsilon} - \mathbf{e}^T\mathbf{E} \\ \mathbf{D} &= \mathbf{e}\boldsymbol{\varepsilon} + \overset{=}{\in}\mathbf{E} \end{aligned} \tag{2}$$

In (2), $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the five component stress and deformation vectors, corresponding to the plane stress situation, $\mathbf{e}$ is the piezoelectric coefficient matrix, $\mathbf{E}$ and $\mathbf{D}$ are the electric field and electric displacement vectors, respectively, $\overset{=}{\in}$ is the dielectric matrix and $\mathbf{Q}$ is the elastic material coefficient matrix which is expressed in terms of the following non dimensional elastic parameters:

$$\begin{aligned}
\alpha_2 &= 4-4E_2/E_1 \\
\alpha_3 &= 1+(1-2\nu_{12})E_2/E_1-4\alpha_0\,G_{12}/E_1 \\
\alpha_4 &= 1+(1+6\nu_{12})E_2/E_1-4\alpha_0\,G_{12}/E_1 \\
\alpha_5 &= (\alpha_1-\alpha_4)/2 \\
\alpha_8 &= 4(G_{13}+G_{23})\alpha_0/E_1 \\
\alpha_9 &= 4(G_{13}-G_{23})\alpha_0/E_1
\end{aligned} \tag{3}$$

where $\alpha_0 = 1-\nu_{12}^2\,E_2/E_1$.

Applying a rotational transformation from the principal material axes (x_1, x_2, x_3) to the global reference system (x, y, z), one obtains the constitutive equations in this later coordinate system [1].

For a laminate made of several composite and piezoelectric material laminae, the constitutive equations are obtained after integration is carried out in the laminate thickness coordinate. The equations of motion for free vibration are then obtained using the finite element method through an eight nodded serendipity plate element with nine degrees of freedom per node, corresponding to the terms in the expansion of the displacement field (1). As for the piezoelectric degrees of freedom, the electric potential variation is constant within each element's piezoelectric layers. With these considerations in mind, one arrives at the following equation of motion at the element level:

$$\begin{bmatrix} \mathbf{M}_{uu}^e & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}} \\ \ddot{\boldsymbol{\varphi}} \end{Bmatrix}^e + \begin{bmatrix} \mathbf{K}_{uu}^e & \mathbf{K}_{u\phi}^e \\ \mathbf{K}_{u\phi}^{eT} & \mathbf{K}_{\phi\phi}^e \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\varphi} \end{Bmatrix}^e = \mathbf{0} \tag{4}$$

where $\mathbf{u}^e$, $\ddot{\mathbf{u}}^e$, $\boldsymbol{\varphi}^e$ and $\ddot{\boldsymbol{\varphi}}^e$ are mechanical degrees of freedom and corresponding accelerations, element electric potential variation and corresponding second time derivatives, respectively. $\mathbf{M}_{uu}^e$ and $\mathbf{K}_{uu}^e$ are the element mass and stiffness matrices, respectively, corresponding to purely mechanical behaviour, while $\mathbf{K}_{\phi\phi}^e$ is the piezoelectric stiffness matrix and $\mathbf{K}_{u\phi}^e$ is the stiffness matrix that corresponds to the coupling between the mechanical and the piezoelectric effects

Still at the element level, the electric degrees of freedom are condensed and considering harmonic vibrations, one obtains the eigenvalue problem:

$$(\mathbf{K}^{*e}-\lambda_i^e\mathbf{M}_{uu}^e)\mathbf{u}_i^e = \mathbf{0} \tag{5}$$

where $\mathbf{u}_i^e$ is the element eigenvector corresponding to the eigenvalue λ_i^e and $\mathbf{K}^{*e} = \mathbf{K}_{uu}^e - \mathbf{K}_{u\phi}^e\mathbf{K}_{\phi\phi}^{e-1}\mathbf{K}_{u\phi}^{eT}$ is the condensed element stiffness matrix.

The global equilibrium equation is then obtained through assembly of the element equations:

$$(\mathbf{K}^* - \lambda_i\mathbf{M})\mathbf{u}_i = \mathbf{0} \tag{6}$$

where $\mathbf{K}^*$ and $\mathbf{M}$ are the global stiffness and mass matrices and $\mathbf{u}_i$ are the system eigenvectors corresponding to the eigenvalues λ_i.

In order to minimize errors associated with modeling boundary conditions, we are only concerned with completely free plates, thus a shift is applied to the global stiffness matrix in order to ensure positive definiteness [16].

PARAMETER ESTIMATION TECHNIQUES

The two proposed parameter estimation techniques are described next. Both of these techniques rely on an experimental response of the active plate structure in the form of undamped natural free vibration frequencies. The gradient base optimization technique is an iterative procedure to solve the inverse problem, while the metamodeling technique approximates the inverse problem through an ANN metamodel.

Gradient Based Optimization

In this approach, the parameter estimation technique consists on minimizing the difference between the response of the physical system and the finite element numerical model that simulates the system response as a function of the elastic and piezoelectric coefficients. The response consists of a set of natural frequencies of free vibration of the plate, which are measured experimentally and then used to fit the corresponding response of the numerical model, thus determining the parameters for the best fit.

The error estimator used in this work is of weighted least squares type:

$$\Phi = \sum_{i=1}^{I} \mathrm{w}_i \left(\frac{\tilde{\lambda}_i - \lambda_i}{\tilde{\lambda}_i} \right)^2 \tag{7}$$

where $\tilde{\lambda}_i$ are the experimental eigenvalues, w_i are weights used to express the confidence level in each one of the experimental eigenvalues, and I is the total number of eigenfrequencies used.

The problem is stated as the constrained minimization of the error estimator (7), where the constraints are imposed in order to insure positive definiteness of the constitutive elastic matrices of all materials:

$$\begin{aligned} \min \quad & \Phi(\mathbf{b}) \geq 0 \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{b}) \leq \mathbf{0} \\ & \mathbf{b}^l \leq \mathbf{b} \leq \mathbf{b}^u \end{aligned} \tag{8}$$

In Equation (8), $\mathbf{b}$ is the vector of design variables, whose components can be non dimensional elastic parameters (3) or piezoelectric constants e_{31} and e_{32} in (2), $\mathbf{g}$ is the constraint set imposed in order for each material's constitutive matrix to remain positive definite and $\mathbf{b}^l$ and $\mathbf{b}^u$ are the side constraints imposed on the design variables.

The numerical optimization technique integrates methods for unconstrained problems, based on Gauss-Newton algorithm with the Feasible Arc Interior Point Algorithm (FAIPA) for constrained optimization [17, 18].

Due to the different order of magnitude of the sensitivities of the eigenfrequencies to different types of possible design variables (elastic, piezoelectric and dielectric), a three phase identification procedure is used [1]. In the first stage, only the elastic parameters associated with the base composite laminate are identified. Next the sensors and actuators are exteriorly bonded to the surfaces of the laminate and the second phase of the identification takes place for the estimation of the elastic parameters of the piezoelectric material, in closed circuit conditions (in order to eliminate the piezoelectric effect). Finally, the third phase consists in identifying only the piezoelectric parameters, in open circuit, noting that the dielectric parameters must be determined experimentally before this third phase, using well established procedures described in ASTM D150-98 [19].

Metamodeling Approach

In this approach a two hidden layer ANN is employed to model the inverse problem, as shown in Fig. 3. The inputs to the network are the undamped natural frequencies of free vibration of the plate and the outputs are the elastic or piezoelectric properties of the laminate. If the elastic properties are to be estimated, then the number of output neurons is six, corresponding to the six independent elastic constants. On the other hand, if the parameters to be estimated are the piezoelectric coefficients, then the number of output neurons is only two.

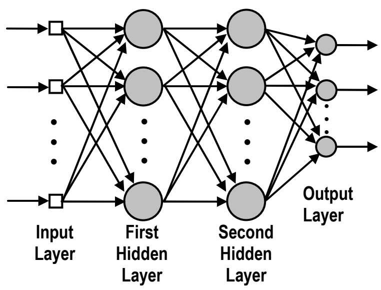

Figure 3. The two hidden-layer ANN model

According to [20, 21] two hidden layers are sufficient to solve this kind of inverse problem and the number of neurons in each hidden layer was chosen to be three times the number of inputs and outputs, respectively [22].

The ANN can be viewed as a non-linear mapping between the frequency space and the space defined by the material parameters to be estimated. Each neuron is the information processing unit represented in Fig. 4.

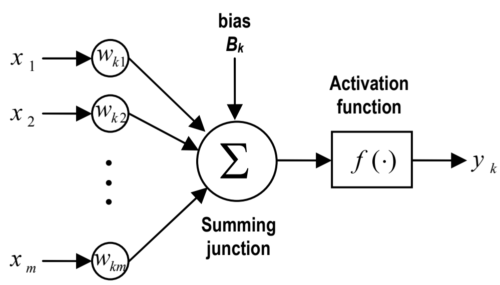

Figure 4. Model of a neuron

where w_{kj} is the weight associated to input x_j of neuron k, B_k is the bias associated with the same neuron and f is the activation function of the neuron. Thus, the output of neuron k can be expressed through:

$$y_k = f\left(\sum_{j=1}^{m} w_{kj} x_j + B_k\right) \tag{9}$$

In this work the activation functions are of the log sigmoid type for neurons in the two hidden layers and of the pure linear type for neurons in the output layer.

The output of the network can be expressed as the non linear mapping (10), where **W** is a matrix of weights and biases, **Y** is the vector of outputs (elastic or piezoelectric parameters) and **X** the vector of inputs (natural frequencies).

$$\mathbf{Y} = \mathbf{f}(\mathbf{W}, \mathbf{X}) \qquad (10)$$

In designing such non linear mapping the parameters to be adjusted are the weights and biases in matrix **W**. This is done through a supervised learning process by feeding the network with pairs of known inputs and corresponding target outputs, produced by the finite element numerical model, and adjusting the weights such that the network produces outputs as close as possible to the known ones. This is defined as an unconstrained minimization problem and the Levenberg-Marquardt method was used to carry out this minimization, using a mean square error estimator.

The training data set for the elastic constants estimation was established by using the concept of orthogonal arrays, thus reducing the full-factorial dimension of these data sets from 5^6 to 25. This corresponds to a L25 orthogonal array for 6 factors (elastic parameters) and 5 levels for each factor [23]. Both inputs and outputs were normalized to unity to avoid saturation.

After training is complete, the ANN can be used to estimate the material constants, by feeding it with the experimental eigenfrequencies. As suggested before, the three phase method was also employed in this approach and the full-factorial dimension of the training data set was employed for the third phase, since there are only two parameters to estimate in this last phase (piezoelectric constants).

NUMERICAL APPLICATION

In this section we present simulations of the two parameter estimation techniques described previously, which are denoted by FAIPA and ANN in this section. The plate is made of a Gr/Epoxy laminate with stacking sequence [90º, 0º, 90º] (w.r.t. horizontal). Each ply of Gr/Epoxy is 1 mm thick. The plate is instrumented with an array of nine pairs of equally spaced 60 mm × 40 mm × 1 mm PZT-4 patches bounded to the top and bottom surfaces of the laminate, as shown in Fig. 5.

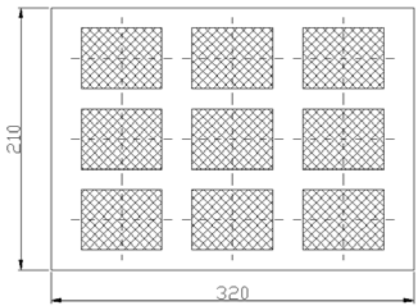

Figure 5. Plate with surface bonded collocated patches (dimensions in mm)

Results of the simulations conducted using the first sixteen natural frequencies and a finite element mesh of 20×20 elements are presented in Table 1 and 2, along with the true properties of each material. All properties are layer properties given in the layer material axes and ρ represents mass per unit volume of each material.

Table 1. Material Properties for Gr/Epoxy

	True	Identified	
		FAIPA	ANN
E_1 [GPa]	132.4	132.4	132.0
E_2 [GPa]	10.8	10.8	10.9
G_{12} [GPa]	5.7	5.7	5.7
G_{13} [GPa]	5.7	5.8	4.5
G_{23} [GPa]	3.6	5.5	2.8
ν_{12}	0.24	0.22	0.30
ρ [kg/m^3]	1578	—	—

Table 2. Material Properties for PZT-4

	True	Identified	
		FAIPA	ANN
E_1 [GPa]	81.3	79.8	82.8
E_2 [GPa]	81.3	79.8	81.3
G_{12} [GPa]	30.6	30.0	31.2
G_{13} [GPa]	25.6	33.2	30.2
G_{23} [GPa]	25.6	32.9	32.3
ν_{12}	0.33	0.30	0.36
e_{31} (N/Vm)	-5.2	-5.4	-4.8
e_{32} (N/Vm)	-5.2	-5.0	-5.9
$\in_{33}$ (10^{-9} F/m)	11.5	—	—
ρ [kg/m^3]	7600	—	—

In Tables 3, 4 and 5 the sixteen natural frequencies and residuals obtained after identification are presented for each one of the three identification phases. The residuals are defined as:

$$r_i = (\tilde{\omega}_i - \omega_i)/\tilde{\omega}_i \times 100 \qquad (11)$$

where $\tilde{\omega}_i$ and ω_i are the experimental and numerical natural frequencies, respectively.

Table 3. Natural frequencies and residuals after identification (Phase 1)

i	$\tilde{\omega}_i$ [Hz]	r_i [%]	
		FAIPA	ANN
1	88.31	0.00	-0.07
2	93.81	-0.05	0.16
3	203.65	0.00	-0.02
4	258.44	-0.03	0.15
5	374.25	-0.01	0.05
6	506.01	-0.01	0.11
7	618.57	-0.01	0.09
8	628.70	0.00	-0.09
9	652.39	0.01	-0.12
10	729.76	-0.00	-0.03
11	835.33	0.02	0.08
12	870.54	-0.02	0.11
13	943.39	0.01	0.10
14	1082.52	-0.00	0.09
15	1247.62	0.03	0.11
16	1349.99	0.05	0.07

Table 4. Natural frequencies and residuals after identification (Phase 2)

i	$\tilde{\omega}_i$ [Hz]	r_i [%]	
		FAIPA	ANN
1	112.52	-0.03	0.10
2	161.29	0.03	-0.14
3	306.93	-0.01	0.11
4	357.03	0.00	-0.11
5	602.56	-0.02	0.06
6	630.57	-0.03	-0.05
7	674.62	0.00	0.02
8	845.68	-0.00	-0.32
9	864.55	-0.10	0.17
10	924.29	-0.02	-0.15
11	1183.96	0.00	-0.05
12	1266.06	0.03	-0.20
13	1333.78	-0.02	-0.06
14	1356.51	0.05	-0.10
15	1619.26	0.06	-0.13
16	1668.57	0.06	-0.01

Table 5. Natural frequencies and residuals after identification (Phase 3)

i	$\tilde{\omega}_i$ [Hz]	r_i [%]	
		FAIPA	ANN
1	112.60	-0.03	0.12
2	161.33	0.02	-0.14
3	307.11	-0.01	0.13
4	357.21	-0.01	-0.10
5	603.36	0.02	0.01
6	630.87	-0.05	-0.04
7	675.43	0.02	-0.01
8	847.47	0.01	-0.25
9	866.39	-0.11	0.18
10	926.29	-0.03	-0.08
11	1185.63	-0.00	0.00
12	1267.72	0.02	-0.18
13	1335.96	-0.03	-0.02
14	1358.75	0.05	-0.09
15	1621.53	0.06	-0.08
16	1671.23	0.09	-0.05

The initial values for material parameters in the gradient based optimization approach are presented in Table 6. These values were chosen to be in the range of typical values for each type of material.

Table 6. Initial values for material parameters in gradient based optimization approach

	Gr/Epoxy	PZT-4
E_1 [GPa]	100.0	100.0
E_2 [GPa]	15.0	100.0
G_{12} [GPa]	6.0	41.0
G_{13} [GPa]	6.0	41.0
G_{23} [GPa]	6.0	41.0
ν_{12}	0.3	0.3
e_{31} (N/Vm)	—	-10.0
e_{32} (N/Vm)	—	-10.0

As for the metamodeling approach, ANN weights and biases were randomly initialized. As a result of this random starting point, optimized weights and biases can vary substantially, although the network always predicts the material parameters correctly with very slight variations.

CONCLUSIONS

Two different approaches to the problem of estimating elastic and piezoelectric properties in active plate structures have been presented along with the results of a simulated identification case study.

From direct observation of the results, one can conclude that both techniques provide reasonable estimates for the parameters, although the ANN technique presents slightly higher residual levels.

Due to the low thickness to length ratio of the test specimens, the transverse shear modulus G_{13} and G_{23} are difficult to identify since the transverse shear is not noticeable.

Regarding the identification of the piezoelectric coefficients, some error propagation occurs because for phases 2 and 3 the identified results from previous phases are used, as in a real identification process. This is especially more evident for the ANN method.

In both approaches, initial values for the design parameters/weights do not influence results in a noticeable way, although this can happen if one does not use enough experimental natural frequencies. Also, when using the ANN approach, care must be taken in order to avoid overfitting by properly adjusting the size of the network.

Comparisons of execution times between the two methods show that ANN takes from about 1.2 to 2.4 times more computer time than FAIPA.

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